hence Eq. (12) may be written in the form

$$S_T = S_S - \delta S$$

where δS is the difference in distance travelled in one cycle time between the two missiles. δS will be small in comparison to S_S, and may be evaluated approximately by binomial expansion, neglecting terms after the third. Hence,

$$\frac{S_T}{S_S} \approx 1 - \frac{(T_P - a - bV_1)}{kV_C b}\,\frac{\dot{m}\tau_P}{2M_0}$$

To simplify this expression, assume $V_1 \approx V_C$ and $a \ll T_P$. Hence,

$$S_T/S_S = 1 - [(k - 1)/k]\delta M/2M_0 \qquad (13)$$

where δM is the total change in mass during the pulse.

From Eq. (13) it may be seen that the fractional loss in distance or velocity of the pulsed missile is proportional to $\delta M/M_0$, which is the fractional change of mass during a pulse. Now k is the ratio T_P/T_C, and when k is large, $(k - 1)/k$ tends to unity. Hence, for large thrust ratios, the fractional loss of velocity is equal to half of the fractional change of mass during a pulse. As k tends towards unity $(k - 1)/k$ tends towards zero, and S_T approaches the values of S_S.

Equation (13) may be shown to be approximately equivalent to

$$S_T/S_S = 1 - \Delta V/2I_{SP}g$$

which is a more convenient form for calculation. For example, a change in velocity during a pulse of 500 ft/sec of a missile with an effective specific impulse of 250 lbf/lbm/sec results in a distance or velocity loss of

$$\delta S/S = 500/(2 \times 250 \times 32) \approx 3\%$$

Hence, in optimization of the velocity increment during a pulse, this effect will tend to favor small increments.

Reference

[1] Ullock, M. H., "Pulse Sustaining of Low-Altitude Cruise Missiles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1220–1221.

Reply by Author to A. J. Cruttenden

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CRUTTENDEN has obtained a closed-form solution for the velocity and range covered by a missile during one cycle of pulse sustaining using a linear variation in drag with velocity. He identifies this solution as accounting for performance loss due to mass change during thrusting. It should be noted, however, that mass change is properly accounted for in the ideal velocity increment used in Ref. 1. The significant conclusion Cruttenden draws is that a large number of pulses is desired for maximum range. However, this conclusion can be misleading from a practical standpoint because of two simplifying assumptions implied by Cruttenden's analysis. These assumptions are: 1) thrust build-up and decay times during each pulse may be neglected; 2) the volume and weight of thermal barriers and ignitors for each pulse may be neglected.

Both of these assumptions are reasonable for a small number of pulses, but are invalid for a large number of short pulses. The method of Ref. 1 was used to conduct an analysis of the effect of the number of pulses on range considering only the weight and volume of barriers and ignitors. The

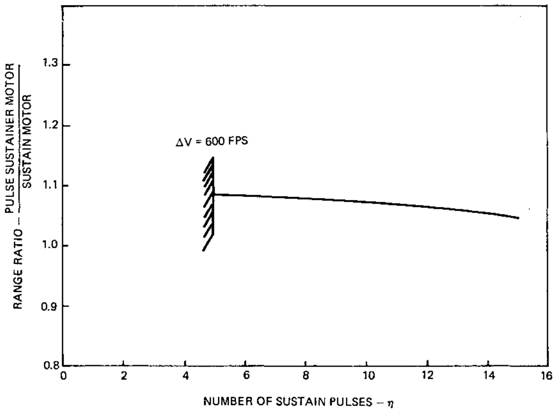

Fig. 1 Effect of the number of sustain pulses on range.

results shown on Fig. 1 indicate that a small number rather than a large number of pulses is desirable for maximum range. Including the effect of thrust build-up and decay would further magnify the decreasing trend with an increasing number of pulse.

Reference

[1] Ullock, M. H., "Pulse Sustaining of Low-Altitude Cruise Missiles," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1220–1221.

Received April 15, 1969.

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Comments on "In Situ Vacuum Testing—A Must for Certain Elastomeric Materials"

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IN reporting their work on the effects of vacuum exposure on the properties of a solid rocket fuel, Mugler et al.[1] state that their data demonstrate that significant changes in properties upon exposure to vacuum can occur even though weight loss is less than 1%. They further note that this finding is contrary to the general guidelines suggested by Muraca and Whittick.[2]

The work reported by Muraca and Whittick[2] is covered in much greater detail in earlier reports[3,4] of research performed on the same program. These contain detailed results of in situ continuous and intermittent tests of a large number of elastomers and in situ constant load tests of several plastic materials. It can be generally concluded from our data that significant changes, other than chemorheological changes, occur only when accompanied by relatively large weight losses (>1%). Loss of weight in such materials is usually due to the irreversible outgassing of low-molecular-weight fragments, breakdown products, residual processing materials, and/or plasticizers or extenders. We also noted that re-exposure to the atmosphere did not result is recovery of properties to pre-exposure values.

In contrast to the essentially homogeneous materials examined in our program, the material studied by Mugler et al. is grossly heterogeneous. A composite solid propellant is highly filled with relatively large particles—particles that are loosely bound in the matrix. A process not occurring, or occurring only to a slight degree, in "homogeneous" materials can account for the behavior noted by the authors. The very small amount of interfacial moisture that influences

Received May 22, 1969.

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mechanical properties of a solid propellant to a high degree can be readily removed on exposure to vacuum, and can also be reabsorbed upon re-exposure to atmosphere. I suggest that in addition to pre-exposure properties, the weight loss noted by the authors was substantially recovered upon re-exposure to the atmosphere. I also suggest that similar results could have been obtained had the specimens been tested in a desiccated environment.

References

[1] Mugler, J. P., Jr. et al., "In Situ Vacuum Testing—A Must for Certain Elastomeric Materials," *Journal of Spacecraft and Rockets*, Vol. 6, No. 2, Feb. 1969, pp. 219–221.

[2] Muraca, R. F. and Whittick, J. S., "Polymers for Spacecraft Applications," Project ASD-5046, JPL Contract 950745, Sept. 15, 1967, Stanford Research Institute.

[3] Fishman, N., in "Space Environment Effects on Polymer Materials," Project ASD-4257, JPL Contract 950324, May 1965, Stanford Research Institute, pp. 37–50.

[4] Fishman, N., in "Polymers for Spacecraft Hardware: Materials Characterization," Project ASD-5046, JPL Contract 950745, Dec. 1966, Stanford Research Institute, pp. 51–73.

Reply by Authors to N. Fishman

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The main purpose of our Note was to show that vacuum-induced changes in the engineering properties of certain elastomeric materials must be determined by measuring the properties in the vacuum environment (in situ). This approach is in contrast to two other approaches in common use: 1) measuring the engineering properties before and after exposure to the vacuum environment, and/or 2) measuring the vacuum weight loss and assuming that the magnitude of the weight loss is indicative of changes in engineering properties. Our results showed that neither of the latter two approaches is valid for the materials studied.

The complete results of our study on the composite solid propellant are presented in Ref. 1, which describes a phenomenological model for the behavior. The analysis[1] indicates that vacuum exposure removes interfacial moisture that results in the observed property changes. Thus, our results are in accord with Fishman's comments on moisture effects. Reference 1 also describes tests of samples in a desiccated environment (dry nitrogen) and the results show that, at a given storage time, the changes in mechanical properties for the samples stored in vacuum were substantially greater than for the samples stored in a desiccated environment.

The authors recommend that in situ engineering properties measurements be used to evaluate spacecraft materials rather than weight loss or other peripheral measurements.

Reference

[1] Greenwood, L. R., "The Effect of Vacuum on the Mechanical Properties of a Solid Rocket Propellant During Space Storage," thesis, May 1967, Virginia Polytechnic Institute, Blacksburg, Va.; University Microfilms, Order 68-2047.

Received July 10, 1969.

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Comments on "Wobble-Spin Technique for Spacecraft Inversion and Earth Photography"

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Although the results of a recent paper[1] by Beachley and Uicker are correct for the assumptions made, it should be emphasized that the concept cannot be reasonably implemented for the stated application. The paper clearly states that the wobble-spin technique is a workable concept "if certain prescribed spacecraft moment-of-inertia relationships are maintained" but then fails to consider the devastating effect of a small deviation from those mass property constraints. It is instructive, in terms of feasibility, to consider the performance of the proposed system in the presence of a small deviation from the idealized mass properties. Their Eqs. (A11–A13) are used as published;

$$I_{11}\dot{\omega}_1 = -J_0\dot{\Omega} + (I_{22} - I_{33})\omega_2\omega_3 \tag{A11}$$

$$I_{22}\dot{\omega}_2 = -J_0\Omega\omega_3 - (I_{11} - I_{33})\omega_1\omega_3 \tag{A12}$$

$$I_{33}\dot{\omega}_3 = J_0\Omega\omega_2 + (I_{11} - I_{22})\omega_1\omega_2 \tag{A13}$$

Instead of rewriting the equations at once with the oversimplifying constraint $I_{22} = I_{33}$ (as in Ref. 1), it is better to solve the general equations for small deviations from the initial conditions. Consider the wheel accelerating period to be very short, after which the wheel runs at constant speed Ω specified by their Eq. (A23). The unsymmetrical spinning spacecraft may be conveniently analyzed by introducing a factor k as in Ref. 2, where

$$k^2 = I_{22}(I_{22} - I_{33})/I_{11}(I_{11} - I_{33})$$

and

$$\omega = \omega_1 + ik\omega_2$$

Then, considering $|\omega| \ll 1$ and $\omega_1 t \ll 1$, the e_3 axis remains near the angular momentum vector and Eqs. (A11–A13) reduce to

$$\begin{aligned} \dot{\omega}_1 - \Omega_1\omega_2 &= -J_0\dot{\Omega}/I_{11} \\ \dot{\omega}_2 + \Omega_2\omega_1 &= -J_0\Omega p/I_{22} \end{aligned} \tag{1}$$

where

$$\Omega_1 = p\left(\frac{I_{22} - I_{33}}{I_{11}}\right) \quad \Omega_2 = p\left(\frac{I_{11} - I_{33}}{I_{22}}\right)$$

$$p = \omega_3 \approx \text{const}$$

Equations (1) become

$$\dot{\omega} + i\Omega_n\omega = -\dot{h}/I_{11} - ikhp/I_{22} \tag{2}$$

where $\Omega_n = (\Omega_1\Omega_2)^{1/2}$, and $h = J_0\Omega$ = angular momentum of reaction wheel. The solution of Eq. (2) for $t < t_1$ (where t_1 = wheel acceleration period) is

$$\omega = \omega(0)e^{-i\Omega_n t} + i(1 - e^{-i\Omega_n t})(1/I_{11}\Omega_n - kp/I_{22}\Omega_n^2)\dot{h} - (kp\dot{h}/I_{22}\Omega_n)t \tag{3}$$

The nutation frequency Ω_n is near zero, so that $\Omega_n t_1 \ll 1$ (where t_1 is the wheel accelerating period; $t_1 \ll 1$), and with $\omega(0) = 0$ the spacecraft motion at the end of the wheel accelerating period is

$$\omega(t_1) \approx -\dot{h}t_1/I_{11} = -J_0\Omega/I_{11} \tag{4}$$

Received June 9, 1969.

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